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Herschel's period, there have been evidently 28 rotations: dividing 286.8 by 28 we have $10^h 14^m 24^s$ for the period of a single rotation, differing two tenths of a second from the determination of Prof. Hall.

Again taking the Washington observations of December 7th $6^h 18^m$, and of January 2nd $7^h 9^m$, both by Prof. Hall, and the first and last in the table, and we have an interval of 624.85 hours with 61 rotations, giving for each rotation a period of $10^h 14^m 24^s$, coinciding with the previous result, and differing from the computation of Professor Hall by two tenths of a second.

[In comparing the period derived from the observed interval between a single pair of observations with that from the adjusted mean of a number of independent observations, the correction, Δt , resulting from the motions of the Earth and Saturn during the observed interval, should have been applied: besides, the quotients in the two cases presented above are respectively, without the correction alluded to, $10h 14m 34s$ and $10h 14m 36s$, instead of $10h 14m 24s^*$ in each case as given above. But even this approximate coincidence must be regarded as accidental, as it is obviously impossible for an observer to determine, by a single observation, the exact time when the center of the spot coincides with the center of the disk of the planet. This fact is recognized in all the observations recorded, as none of the observers have registered their observations in fractions of a minute. Moreover the three contemporaneous observations by Professors Hall, Newcomb and Eastman, who may be assumed to be equally skilful observers, give for the least and greatest difference, respectively, one and five minutes; which shows that the true result can only be obtained with exactness from the adjusted mean of many independent observations.—Ed.]

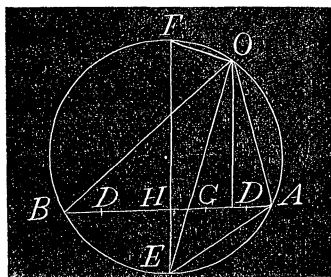
REMARKS ON PROBLEM 137, BY GEORGE EASTWOOD. — The solution given at page 26 of the ANALYST, Vol. IV, may be greatly simplified by assuming the given point D to be at the foot of a perpendicular from A upon the given line BC .

In the annexed diagram, let $AB = a$ a given line $= l$, $\angle ABD = \alpha$ a given angle $= \omega$, D the given point, and OD the perpendicular from O on $AB = p$, a given line.

Put $\angle OAB = \theta$; then $AD = p \cot \theta$

$$BD = p \cot(\omega - \theta)$$

$$\therefore p \cot \theta + p \cot(\omega - \theta) = l.$$



*Mr Evans' result is obtained by limiting the quotient in both cases to two decimals.

$$\therefore \cot \theta + \frac{\cot \theta \cot \omega + 1}{\cot \theta - \cot \omega} = \frac{l}{p},$$

an equation of the second degree in $\cot \theta$.

$$\therefore \cot \theta = \frac{l}{2p} \pm \frac{1}{2p} \sqrt{(4p \cot \omega - 4p^2 + l^2)}.$$

These values of $\cot \theta$ show that the point D may be posited at the same distance from B that it is from A .

The question, under this additional data, is equivalent to the following:

Given the base, the vertical angle, and the perpendicular from the vertical angle upon the base, to construct the triangle.

Analysis:—Suppose the thing done, AOB the required triangle, circumscribed by the circle $AOFBE$, O the given vertical angle, AB the given base, and OD the given perpendicular. Bisect the base in H , and draw the diameter FHE parallel to OD . Join OF meeting AB in G , and draw AE . Then, since AH is given, and the angle $HAE = AOE = \frac{1}{2}$ given vertical angle, $\therefore AE, HE$, and EF are given. Also, since the line OE bisects the angle AOB , the rectangle under $AO.OB = OE.OG = (OG + GE)OG = OG^2 + OG.GE = EF.OD$, and by a known property $(OG + GE)GE = GE^2 + OG.GE = AE^2$. That is $OG^2 + 2OG.GE + GE^2 = EF.OD + AE^2 =$ sum of two given magnitudes. Make $EF.OD =$ a square $= M^2$, and $M^2 + AE^2 = N^2$, then $OG^2 + 2OG.GE + GE^2 = N^2 =$ a given square. Therefore $OG + GE =$ a given line.

But $(OG + GE)GE = AE^2$, therefore GE , and consequently OG , are both given; therefore $OG.GE =$ a given rectangle.

Again, $BG.AG = (AB - AG)AG = AB.AG - AG^2 = OG.GE =$ a given magnitude. That is, we are required to take a line from a given line, so that the rectangle under the two parts shall be equal to a given rectangle—a well known geometrical problem. Now AG being thus determined, BG will be known. But $AO : OB :: AG : BG :: 1 : n$, a given ratio. Therefore $nAO = OB$, and $nAO^2 = EF.OD = M^2$, as above. Therefore AO and OB , the sides of the triangle are known. Whence this

Construction: Upon AB the given base, describe the segment of a circle capable of containing the given angle. At D , the given point, erect the perpendicular DO , meeting the segment in O : join AO, BO , then AOB is the required triangle. The demonstration is manifest from the analysis.

When the lines AO, OB, BA , and OD are arcs of great circles, then, putting angle $O = \omega$, angle $AOD = \alpha$, $OD = p$, and $AB = \lambda$, we have

$$\tan AD = \sin p \tan \alpha, \quad \tan BD = \sin p \tan (\omega - \alpha),$$

$$\therefore AD + BD = \tan^{-1} \sin p \tan \alpha + \tan^{-1} \sin p \tan (\omega - \alpha) = \lambda. \quad (1)$$

Therefore, taking tangent function of both sides of (1), we get

$$\frac{\sin p \tan \alpha + \sin p \tan (\omega - \alpha)}{1 - \sin^2 p \tan \alpha \tan (\omega - \alpha)} = \tan \lambda,$$

or, by dropping functional characteristics of α , ω and λ ,

$$\sin p \left(\alpha + \frac{\omega - \alpha}{1 + \alpha \omega} \right) \div \left(1 - \sin^2 p \frac{\omega \alpha - \alpha^2}{1 + \alpha \omega} \right) = \lambda. \quad . \quad . \quad . \quad (2)$$

By expansion and reduction, (2) becomes

$$(\omega \sin p - \lambda \sin^2 p) \alpha^2 - \lambda \omega \cos^2 p \alpha = \lambda - \omega \sin p. \quad . \quad . \quad . \quad (3)$$

The two roots of α in (3) are

$$\frac{\omega \lambda \cos^2 p}{2(\omega \sin p - \lambda \sin^2 p)} + \frac{[4(\lambda - \omega \sin p)(\omega \sin p - \lambda \sin^2 p) + \lambda^2 \omega^2 \cos^2 p]^{\frac{1}{2}}}{2(\omega \sin p - \lambda \sin^2 p)}, \text{ and}$$

$$\frac{\omega \lambda \cos^2 p}{2(\omega \sin p - \lambda \sin^2 p)} - \frac{[4(\lambda - \omega \sin p)(\omega \sin p - \lambda \sin^2 p) + \lambda^2 \omega^2 \cos^2 p]^{\frac{1}{2}}}{2(\omega \sin p - \lambda \sin^2 p)},$$

which shows, as in a plane, that the point D may be posited at the same distance from B that it is from A .

This problem, in its most general form, has a somewhat remarkable history. A general solution may be found in Newton's Universal Arithmetic, prob. 24; and in Gergonne, *Annales de Mathematiques*, tom. 10, p. 205.

ON ADJUSTMENT FORMULAS.

BY E. L. DE FOREST.

WHEN a series of observed numbers shows irregularities resulting from errors of observation, the simplest mode of correcting them is by graphical construction, plotting the terms of the series on paper as ordinates to a curve, drawing a smooth curve through the points thus obtained so as to coincide with them as nearly as may seem best, and then measuring the ordinates thus corrected. This will often give results accurate enough for practical purposes. It also has the advantage of bringing the conditions of the problem together before the eye, so that graphical construction will always be a valuable adjunct and preliminary to the use of other methods which may be expected to give a higher degree of accuracy.

Of the analytical methods for adjusting a series, the simplest and most easily applied by persons of moderate mathematical knowledge is that which presupposes that the terms of the given series are equidistant, or if not, that they have been reduced to equidistant ones either graphically or by simple interpolation, and then adjusts the middle term u_0 of any group of